III. Fitting Theoretical and Experimental Results

Resolving the subtle differences between theoretical and experimental parameters is a challenging prospect. For this reason, the proper fitting of theoretical and experimental results deserves clarification. The sole purpose of this appendix is to outline the tedious (and sometimes tenuous) procedures necessary to accurately compare the analytic model and experimental results. Measurement constraints and physical arguments are made which elucidate the various differences between theory and experiment and compensation for these differences is provided.

A.3.1 Measured OSNR vs. Theoretical OSNR

It is advantageous to write P_{RB} and N_{ASE} in terms of the average signal power, Rayleigh OSNR and ASE OSNR because of the ease with which these terms can be measured experimentally. By definition

$$OSNR_{ASE(t)} = \frac{P_{ave}}{B_{RBW} N_{ASE}} \tag{A.3.1}$$

$$OSNR_{RB(t)} = \frac{P_{ave}}{P_{RB}} \tag{A.3.2}$$

where P_{ave} is the average optical signal power prior to optical filtering and P_{ASE} and P_{RB} are the average noise powers due to ASE and RB prior to optical filtering. The subscript (t) is included to emphasize the fact that these are theoretical definitions and not necessarily the same as what is actually measured in a laboratory. The experimental techniques to measure $OSNR_{ASE}$ and $OSNR_{RB}$ are quite different. While $OSNR_{RB}$ is straightforward to measure experimentally, $OSNR_{ASE}$ can be a problematic quantity to measure directly. In order to correctly calculate the ASE power spectral density N_{ASE}, care must be taken to relate the measured $OSNR_{ASE}$ to the definitional (i.e. theoretical) $OSNR_{ASE}$. The following analysis demonstrates how to convert between the measured $OSNR_{ASE}$ and the actual N_{ASE} spectral density as it pertains to the analytic model.

A.3.1.1 $OSNR_{RB}$ Measurement

The measurement of $OSNR_{RB}$ is straightforward. First, the average signal power at the input to the optical preamplifier is measured with a power meter. Then, the signal is turned off and the Rayleigh generating signal is turned on. The Rayleigh backscattered light from the fiber spool is passed through a circulator and the total power is measured at the input to the preamplifier. The ratio of these two measurements gives the $OSNR_{RB}$.

A.3.1.2 $OSNR_{ASE}$ Measurement

The commonly accepted procedure of using an optical spectrum analyzer (OSA) to measure $OSNR_{ASE}$ is outlined by Fig. A.3.1. By measuring the peak level, P_s, of the signal spectrum and the noise floor, P_{ASE}, on an OSA immediately *after* the preamplifier, the $OSNR_{ASE}$ can be extracted for a given resolution bandwidth (traditionally chosen to be 0.1 nm). The ratio of P_s and P_{ASE} gives:

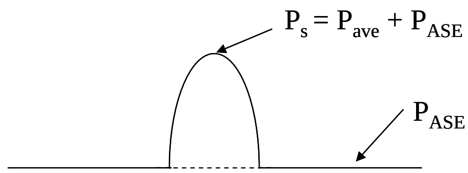

Fig. A.3.1. Illustration of the OSA OSNR measurement. The ASE is assumed to have a flat spectral density and the signal peak consists of both ASE and signal.

$$OSNR_{ASE(m)}(t) = \frac{\gamma_{\mathrm{mod}} P_{ave} + P_{ASE}}{P_{ASE}} = ONSR_{ASE(t)} + 1 \qquad \text{(A.3.3)}$$

Rearranging (A.3.3) yields

$$OSNR_{ASE(t)} = \frac{OSNR_{ASE(m)} - 1}{\gamma_{\mathrm{mod}}} \qquad \text{(A.3.4)}$$

where γ_{mod} is a modulation dependent scaling factor (γ_{mod} ε [0,1]). The subscript (m) indicates the measured $OSNR_{ASE}$ and (t) indicates the theoretical $OSNR_{ASE}$. The above equation is valid as long as P_{ASE} is constant across the band being measured.

The inclusion of the term γ_{mod} stems from the fact that the signal spectrum being measured might be larger than the resolution bandwidth of the OSA. If this is the case, some finite portion of the signal will lie outside the bandwidth of the OSA and will not contribute to the peak power reading, P_s, causing P_{ave} to be underestimated This underestimation of P_{ave} can be calibrated out by calculating the amount of signal power which is clipped by the OSA filter. Table A.3.1 lists the percentage of power which lies within a 0.1 nm bandwidth for NRZ, RZ and DB. The majority of the power lies within 0.1 nm for NRZ and DB while only about 73.6% of the energy lies within 0.1 nm for RZ.

Table A.3.1.
Scaling coefficients for the analytic model

Modulation Format	γ_{mod} ($P_{0.1nm}/P_{total}$)	α_{mod}	F_{ISI} (10 GHz 4th Order Bessel)
NRZ	0.960	0.5	1
RZ	0.736	0.25	0.8122
DB	0.993	0.578	0.9227

A.3.1.3 $OSNR_{ASE}$ and N_{ASE}—No RB

The experimental techniques used to measure $OSNR_{ASE}$ and $OSNR_{RB}$ were described in the previous section. The next step is to use these measured values of $OSNR_{ASE(m)}$ and $OSNR_{RB(m)}$ to calculate the power spectral density of the ASE noise, N_{ASE}.

As mentioned earlier, the measured $OSNR_{ASE}$ value is performed on an OSA and is quoted for a specific resolution bandwidth. By definition the total measured noise power P_{ASE} is:

$$P_{ASE} = 2B_{RBW}N_{ASE} \tag{A.3.5}$$

where B_{RBW} is the resolution bandwidth of the OSA resolution filter. The factor of 2 refers to the fact that P_{ASE} is the total noise power in *both* polarizations while N_{ASE} is the power spectral density of a *single* polarization. Plugging (A.3.1) and (A.3.5) into (A.3.4) and solving for N_{ASE} yields:

$$N_{ASE} = \frac{\gamma_{mod}P_{ave}}{2B_{RBW}(OSNR_{ASE(m)} - 1)} \tag{A.3.6}$$

This expression is valid so long as ASE is the dominant noise source.

A.3.1.4 $OSNR_{ASE}$, $OSNR_{RB}$ and N_{ASE}—With RB

If RB is present in the link, (A.3.6) needs to be modified. In particular, we need to account for the fact that the peak signal power, P_s, has an additional contribution as shown in Fig. A.3.2. The difficulty posed by including RB into the calculation of NASE comes from the fact that the EDFA preamplifier's noise performance depends on the input power. Namely

$$OSNR_{out} = \frac{1}{\dfrac{1}{OSNR_{in}} + \dfrac{NFh\nu\Delta f}{P_{in}}}. \qquad \text{(A.3.7)}$$

Here, NF is the noise figure of the EDFA, h is Planck's constant, ν is optical frequency, Δf is the resolution bandwidth of measurement, and P_{in} is the input power into the amplifier. If it can be assumed that $OSNR_{in}$ >> Pin/NFhνΔf, (23) collapses to the well known relation:

$$OSNR_{out} = \frac{P_{in}}{NFh\nu\Delta f}. \qquad \text{(A.3.8)}$$

Rewriting (A.3.8) in dB yields

$$OSNR_{out} = 58 + P_{in} - NF \qquad \text{(A.3.9)}$$

where it is assumed that h = $6.626\text{x}10^{-34}$, ν = 193.1 GHz, and Δf = 0.1 nm. If it is assumed that NF does not fluctuate significantly for small changes in P_{in}, (24) demonstrates that $OSNR_{out}$ (i.e. $OSNR_{ASE}$) changes linearly with Pin. Therefore, dB changes in P_{in} cause dB changes in $OSNR_{out}$.

Using (A.3.8) and (A.3.9), it is now possible to include the effect of RB on the $OSNR_{ASE}$ measurement. Let us assume for the moment that we have measured the

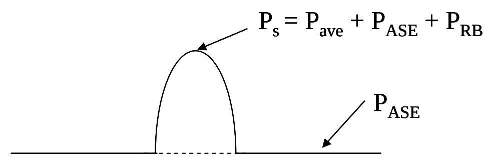

Fig. A.3.2. Illustration of the OSA OSNR measurement in the presence of RB noise. The measured signal power consists of average signal power, RB and ASE.

$OSNR_{ASE}$ value for a certain P_{in} when the Rayleigh signal is turned OFF. Then, we turn ON the Rayleigh signal and measure the $OSNR_{RB}$ value using the method described previously. The question becomes, what is the new $OSNR_{ASE}$ value when we have both the signal and RB present at the input to the preamplifier? Assuming that the overall RB average power is not large enough to change the NF of the amplifier (true as long as P_{RB} is small compared to P_{in}) it is clear from (A.3.9) that the new $OSNR_{ASE}$ will improve slightly by a factor of P_{RB}. The physical explanation of this is straightforward: because there are *more* photons at the input of the amplifier to cause *stimulated* emission, *less spontaneous* photons will be generated during amplification and thus the $OSNR_{ASE}$ will improve. This explanation is valid as long as the EDFA is operated in the linear regime such that gain saturation is avoided.

The fact that the new $OSNR_{ASE}$ will increase by a factor of P_{RB} when RB is present can be incorporated into (A.3.6) with the following equation:

$$N_{ASE} = \frac{\gamma_{\mathrm{mod}} P_{ave}}{2B_{RBW}\left(OSNR_{ASE(m)}\left(1+\dfrac{1}{OSNR_{RB(m)}}\right)-1\right)} \qquad \text{(A.3.10)}$$

Clearly, if $OSNR_{RB(m)}$ is large, the impact of Rayleigh on the ASE noise performance of the preamplifier is negligible and (A.3.10) collapses to become (A.3.6). However, if P_{RB} is on the same order as P_{ave} (i.e. $OSNR_{RB} \sim 1$), the actual N_{ASE} level will be less than expected had (A.3.6) been used.

It should be highlighted that (A.3.10) is only valid when $OSNR_{in}$ in (A.3.7) is large. If this is not the case, a more rigorous approach to calculate N_{ASE} when RB is present should be taken by also including the effect of $OSNR_{in}$. Luckily, the experiments described in this paper do not require such a rigorous approach. As it turns out, for experiments where $OSNR_{RB(m)}$ was low, $OSNR_{in}$ tended to be very large (>20 dB) making (A.3.10) valid. For experiments where $OSNR_{in}$ was small, $OSNR_{RB(m)}$ tended to be large meaning the effect of RB could be neglected making (22) valid.

A.3.2 Peak Power, Average Power and Extinction Ratio

The analytic model treats the peak power, P_1, of the marks while experimental measured results give average power, P_{ave}. This difference must be compensated. Additionally, real signals have some non-negligible amount of energy in the spaces due to finite extinction ratio. These considerations cause confusion when calculating signal dependent noise variances as described by (2.4) and (2.6). The following expressions relate this peak power to the mark to the average power, the "mark density" scaling term, and the extinction ratio of the signal.

$$P_1 = \frac{P_{ave}}{\alpha_{\text{mod}}} - P_0 = \frac{P_{ave}}{\alpha_{\text{mod}}(1+e_x)} \qquad \text{(A.3.11)}$$

where P_0 is the excess light which leaks into the space period and e_x is the extinction ratio coefficient defined by P_0/P_1. The coefficient α_{mod} is a scaling factor necessary to properly adjust P_{ave} for the different modulation types under investigation. Physically, the inclusion of α_{mod} is manifested by the fact that different modulation schemes have different pulse widths and hence different *effective* mark densities. For a given input power, the total average power at the output modulator will completely depend on which modulation scheme is being considered. Table A.3.1 lists the α_{mod} values for NRZ, RZ, and DB. It is interesting to note that DB has an α_{mod} coefficient greater than 0.5, despite the assumption of 50% mark density (i.e. equal number of marks and spaces). The cause of this is twofold. First, the electrical low pass filtering of the original bit sequence yields marks which are broader than there allotted time slots. This gives rise to mark energy which "spills over" into adjacent space time slots. Second, the significant low pass filtering also generates zero ripples between adjacent spaces causing additional energy to accumulate in the spaces.

A.3.3 Analytic Model—In Terms of Measured Parameters

The derivations described in the previous sections enable the four variance equations to be written entirely in terms of readily measured parameters. Replacing P_1, N_{ASE} and P_{RB} with the measured parameters of $OSNR_{ASE(m)}$, $OSNR_{RB(m)}$, e_x, α_{mod}, and γ_{mod} yields:

$$\sigma^2_{sig-ASE(MARK)} = \frac{\dfrac{2R_D^2 P_{ave}{}^2 \gamma_{mod} B_e}{\alpha_{mod}(1+e_x)B_{RBW}\left(OSNR_{ASE(m)}\left(1+\dfrac{1}{OSNR_{RB(m)}}\right)-1\right)}}{\sqrt{1+\dfrac{B_s^2}{B_o^2}+\dfrac{B_s^2}{4B_e^2}}\sqrt{1+\dfrac{2B_s^2}{B_o^2}+\dfrac{4B_e^2}{B_o^2}\left(1+\dfrac{B_s^2}{B_o^2}\right)}} \qquad \text{(A.3.12)}$$

$$\sigma^2_{ASE-ASE(MARK)} = \frac{\dfrac{R_D^2 P_{ave}{}^2 \gamma_{\text{mod}}{}^2 B_e B_o}{\sqrt{2} B_{RBW}{}^2 \left(OSNR_{ASE(m)} \left(1 + \dfrac{1}{OSNR_{RB(m)}} \right) - 1 \right)^2}}{\sqrt{1 + \dfrac{2B_e^2}{B_o^2}}} \quad \text{(A.3.13)}$$

$$\sigma^2_{sig-RB(MARK)} = \frac{\dfrac{2\eta_{pol} R_D^2 P_{ave}{}^2}{\alpha_{\text{mod}}(1+e_x) OSNR_{RB(m)}}}{\sqrt{1+\dfrac{B_s^2}{B_o^2}}\sqrt{1+\dfrac{2B_s^2}{B_o^2}+\dfrac{B_s^2}{2B_e^2}}\sqrt{1+\dfrac{B_s^2}{B_o^2}+\dfrac{B_s^2}{4B_e^2}}} \exp\left(-\frac{\pi \Delta f^2 \left(4 + \dfrac{4B_s^2}{B_o^2} + \dfrac{2B_s{}^2}{B_e{}^2} + \dfrac{B_o{}^2}{B_e{}^2} \right)}{2B_o^2 \left(1 + \dfrac{B_s^2}{B_o^2} \right) \left(2 + \dfrac{2B_s^2}{B_o^2} + \dfrac{B_s^2}{B_e^2} \right)} \right) \quad \text{(A.3.14)}$$

$$\sigma^2_{RB-RB(MARK)} = \frac{\dfrac{2R_D^2 P_{ave}{}^2}{OSNR_{RB(m)}{}^2}}{\sqrt{1+\dfrac{B_s^2}{B_o^2}}\sqrt{1+\dfrac{B_s^2}{B_o^2}+\dfrac{B_s^2}{2B_e^2}}} \exp\left(-\frac{2\pi\Delta f^2}{B_o^2 \left(1 + \dfrac{B_s^2}{B_o^2} \right)} \right). \quad \text{(A.3.15)}$$

Eq. (A.3.12-A.3.15) represent the variances of marks. The expressions for the variance of the spaces are similar, except that the signal-ASE and signal-RB terms are multiplied by e_x to account for the lower signal strength in the space.

A.3.4 Q-factor Compensation

The strict definition of Q-factor is

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \quad \text{(A.3.16)}$$

where μ_1 and μ_0 are the average photocurrents for the marks and spaces, respectively. Using (A.3.11), the numerator can be rewritten

$$\mu_1 - \mu_0 = R_D (P_1 - P_0) = \frac{R_D P_{ave} (1 - e_x)}{\alpha_{\text{mod}} (1 + e_x)} \quad \text{(A.3.17)}$$

An implicit assumption of (A.3.11) is that the optical and electrical filters under consideration do *not* affect the means of the marks and spaces. Thus, (A.3.17) is only true when the receiver's optical and electrical bandwidth are much greater than the signal's bandwidth. In practice this will not be true. In fact, it is often advantageous to sacrifice a certain amount of filter induced eye closure in order to reduce the amount of high frequency noise in the receiver. Optimal amounts of electrical filtering which trade increased ISI for reduced high frequency noise lie in the range from 0.6 to 1 times the bit rate, depending on the specific link parameters used (see Chapter 3 results).

Because filter induced eye closure is a deterministic effect dependent on the modulation format and optical and electrical filters, the calculation of Q which includes ISI is straightforward. In [1] the Q-factor equation of (A.3.16) is scaled by the term F_{ISI} ε [0,1] represents the percentage of eye closure experienced by the signal due to filtering. Scaling the Q-factor equation by F_{ISI} yields:

$$Q = \frac{\dfrac{R_D P_{ave}(1-e_x)}{\alpha_{\mathrm{mod}}(1+e_x)} F_{ISI}}{\sigma_1 - \sigma_0} \qquad \text{(A.3.18)}$$

The physical tradeoff between increased ISI and reduced noise on the maximization of Q is obvious from (A.3.18): although the denominator of (A.3.18) will decrease with heavy receiver filtering, at some point the overall Q-factor will drop because F_{ISI} will begin to dominate the numerator of (A.3.18).

For this study, F_{ISI} was numerically determined using VPItransmissionMaker™ (VPI Photonics) by taking the ratio of the mean eye opening after filtering divided by the mean eye opening before filtering. Table A.3.1 shows the

F_{ISI} values found in simulation for NRZ, RZ and Duobinary for various receiver bandwidths.

A.3.5 References

[1] P. J. Winzer and A. Kalmar, "Sensitivity enhancement of optical receivers by impulsive coding," *Lightwave Technology, Journal of*, vol. 17, pp. 171-177, 1999.