I. Statistics of Rayleigh Backscattering

Linear optical crosstalk can be classified into several categories as shown in Fig. A.1.1. The various types of crosstalk are characterized by the mutual coherence between interfering field (x-axis) and the total number of interferers (y-axis). In the limit that the number of interferers approaches infinity and the mutual coherence goes to zero (i.e. interferers are totally uncorrelated), linear crosstalk can be thought of as the summation of a distributed number of statistically independent interferers. RB noise lies in this special case of distributed, incoherent optical crosstalk and the mathematical formalism underlying its treatment has been well documented [1-6].

The physical source of RB is illustrated in Fig. A.1.2. When source light is injected into an optical fiber, sub-wavelength refractive index impurities act as small scattering elements which radiate light. Some of the scattered light enters into non-propagating modes of the fiber and it thus lost to the environment. This non-propagating light accounts for the majority of the loss in the fiber [5]. Some portion of the light, however, is scattered into a backwards traveling mode. Owing to the amorphous spatial configuration of the fiber impurities, the total backwards traveling light is comprised of many independently Rayleigh scattered photons from different sections of the fiber. The resultant field at the input of the fiber appears as colored noise.

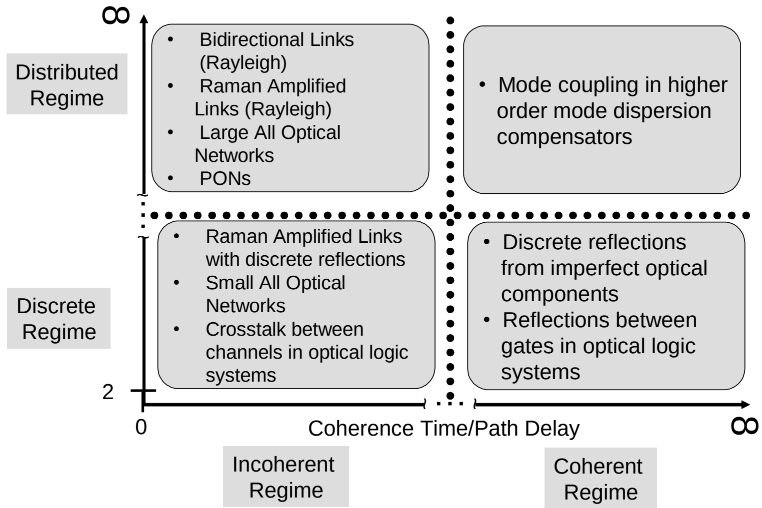

Fig. A.1.1. Overview of the various linear crosstalk regimes. Adapted from [5].

It should be noted that higher order scattering can also occur: the single Rayleigh scattered light can be scattered a second time thus resulting in noise propagating co-directionally with the original signal. In passive fibers (i.e. in the absence of distributed gain), this DRB is negligibly small and is ignored in this work. However, DRB remains an important consideration when designing links which incorporate distributed Raman amplification because it experiences twice as much gain as the original signal. Therefore, DRB grows faster than the signal in the presence of distributed amplifications. The end result is that the total on-off gain of DRAs must be kept to moderate levels to avoid significant DRB penalties [5].

This Appendix provides an overview of the theory behind the modeling of (single) RB noise, specifically regarding the underlying assumptions which allow the

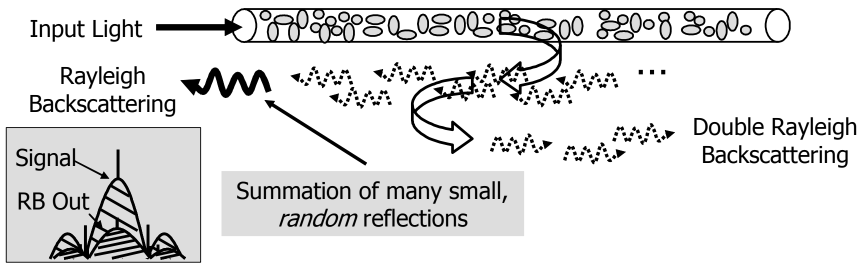

Fig. A.1.2. Physical interpretation of single and double RB. Small inhomogeneities in the fiber generate distributed reflection over many km. The inset shows that the signal and RB have proportional PSDs.

RB field to be treated as a ccg, WSS random process whose ACF is proportional to the of

the original signal light. The following derivation was first offered by Gysel and Staubli in [1, 2] and is reproduced here due to its vital importance.

The input light field is

$$\vec{e_s}(t) = \mathrm{Re}\left\{\vec{E_s}(t)e^{j2\pi\upsilon_0 t}\right\} \tag{A.1.1}$$

Where ν_0 is the optical carrier frequency and $\vec{E_s}(t)$ is the complex amplitude vector of the signal given by

$$\vec{E_s}(t) = \vec{p}E_s(t) = \vec{p}\sqrt{I_0}\gamma(t)e^{j\phi(t)} \tag{A.1.2}$$

Here, $\vec{p}$ is the polarization of the signal field, I_0 is the mean intensity, $\phi(t)$ is the phase noise of the source laser and $\gamma(t)$ is the complex amplitude *modulation*. For simplicity, it will be assumed that polarizations are always co-polarized and that the phase fluctuations of the laser are negligibly small as these modifications do not distract from the main conclusions of this Appendix.

By definition, the ACF of source field is

$$\Gamma_{E_s}(t_1,t_2) = \left\langle E_s(t_1)E_s^*(t_2)\right\rangle = I_0\left\langle \gamma(t_1)\gamma^*(t_2)\right\rangle \quad \text{(A.1.3)}$$

where <·> denotes statistical ensemble averaging. Although the modulation signal, γ(t) is a nonstationary process, the PSD of the modulation is given by the time averaged ACF. Therefore, the PSD of the signal is Fourier transform related to the ACF

$$\overline{\Gamma_{E_s}}(\tau) \xrightarrow{FT} S_{E_s}(f) \quad \text{(A.1.4)}$$

where the over-bar represents time averaging of the ACF in (3).

Upon injection into the fiber, the incident field at any point in the fiber can be written as

$$\overrightarrow{E_i}(t,z) = \overrightarrow{E_s}(t - z/v)e^{-\alpha z/2}e^{j\beta z} \quad \text{(A.1.5)}$$

where b is the propagation constant, v is the group velocity and a is the propagation loss in the fiber.

As described above, small refractive index differences result in portion of the incident light in the fiber to be backscattered. The received backscattered component at the input of the fiber from a scattering section at distance z in the fiber can be written as

$$\Delta\overrightarrow{E_b}(t,z) = \overrightarrow{E_s}(t - 2z/v)e^{-\alpha z}e^{-j2\beta z}\Delta\rho(z) \quad \text{(A.1.6)}$$

where Δρ(z) is the scattering coefficient. Physically, Δρ(z) is related to the average amount of light which will be backscattered by a scattering event. The total RB field will be the superposition of many backscattered components from various locations within the fiber. That is,

$$\overrightarrow{E_b}(t) = \sum_{n=1}^{N} \Delta \overrightarrow{E_b}(t, n\Delta l) \qquad \text{(A.1.7)}$$

In the limit that Δl→0, (A.1.7) can be converted to integral form where

$$\overrightarrow{E_b}(t) = \int_0^L \overrightarrow{E_s}(t - 2z/v)e^{-\alpha z}e^{-j2\beta z}\rho(z)dz \qquad \text{(A.1.8)}$$

The differential backscattering coefficient, $\rho(z) = \lim_{\Delta l \to 0}\left[\frac{\Delta\rho(z)}{\Delta l}\right]$, represents the amount of backscattered light from an infinitesimally small scattering section. *As long as the distance between independent scattering sections is much smaller than the scales over which the phase, amplitude and polarization of the incident field change, ρ(z) will be a zero mean ccg random variable.* Since there are approximately 10 scattering centers/μm of fiber, ccg statistics hold for all practical cases [7].

An alternative expression for the RB field amplitude is given by the convolution of the fiber impulse response with the complex (modulated) source field

$$\overrightarrow{E_b}(t) = \overrightarrow{h_{E_b}}(t) \otimes E_s(t) \qquad \text{(A.1.9)}$$

$$= \frac{v}{2}\int_0^{2L/v} e^{-\alpha v\tau/2}e^{-j\beta v\tau}\rho(v\tau/2)E_s(t-\tau)d\tau$$

where space parameter, z, has been replaced by time, τ, using the relation τ = 2z/v and the fiber impulse response is given by

$$\overrightarrow{h_{E_b}}(t) = \begin{cases} e^{-\alpha v\tau/2}e^{-j\beta v\tau}\rho(v\tau/2), & 0 \le t \le 2L/v \\ 0, & otherwise \end{cases} \qquad \text{(A.1.10)}$$

The PSD of the RB field can be computed by first calculating the ACF

$$\Gamma_{E_b}(t_1,t_2) \equiv \left\langle E_b(t_1)E_b^*(t_2)\right\rangle \tag{A.1.11}$$

Calculation of (A.1.11) is greatly simplified by making use of the fact that $h_{E_b}(t)$ and $E_s(t)$ are statistically independent and that ρ(vt/2) is *delta correlated* owing to its zero mean ccg statistics. After some work, the following relation is obtained,

$$\Gamma_{E_b}(t+\tau,t) = h_{I_b}(t) \otimes R_{E_s}(t+\tau,t) \tag{A.1.12}$$

With

$$h_{I_b}(t) = \begin{cases} \dfrac{v\alpha_s S}{2} e^{-\alpha v \tau}, & 0 \le t \le 2L/v \\ 0, & otherwise \end{cases} \tag{A.1.13}$$

where α_s is the Rayleigh scattering loss and S the recapture fraction of scattered light.

Careful analysis of (A.1.12) and (A.1.13) reveals that the modulated (and therefore nonstationary) input light is heavily lowpass filtered by the impulse response of the fiber. In essence, the fiber acts as a sort of “capacitive” element for the backscattered light resulting in response times on the order of ms (i.e. kHz). It is this buildup of light energy within the first 10-30 km of fiber which allows the resultant RB field to be treated as a WSS random process in most practical situations.

For the majority of cases, the signal will be digitally modulated and thus cyclostationary in the wise sense. Most often, the bit duration T_s will be significantly smaller than the fiber flight time. Moreover, the loss of the fiber will be relatively small such that the overall field amplitude will be constant over the bit period. The physical result of these relationships is that the total RB field will not depend on the actual bit patterns transmitted by the source because the fiber lowpass cutoff is significantly smaller than the data rate. Hence, the fiber has an averaging effect on the RB field. Under these conditions, the RB field will become WSS and

$$\Gamma_{E_b}(\tau) = \kappa\overline{\Gamma_{E_s}}(\tau) \tag{A.1.14}$$

where κ is a scaling term related to the mean backscattered intensity. From the Wiener-Khinchin theorem, the PSD and the ACF are Fourier Transform related such that

$$\Gamma_{E_b}(\tau) \xrightarrow{FT} S_{E_b}(f) \tag{A.1.15}$$

$$S_{E_b}(f) \propto S_{E_s}(f) \tag{A.1.16}$$

Experimental verification of the proportional relationship between the signal and RB PSDs is shown in Fig. 2.7.

Lastly, conditions can be identified which allow for the RB field to be treated as an ergodic ccg random process. Assuming that the coherence time of the laser is much smaller than both the fiber flight time and that losses are sufficiently small such that the amplitude of the signal is roughly constant over a coherence interval, the RB field can be thought of as the accumulation of a large number of independent, evenly distributed scattering sections. By the central limit theorem, the RB field will asymptotically approach ccg statistics.

To summarize, the RB noise field can be treated as a complex circular Gaussian (ccG), wide sense stationary (WSS) random process whose autocorrelation function (ACF) is proportional to the of the original signal light if the following physical assumption hold:

1. The coherence of the source is much smaller than the effective length of fiber

contributing to RB noise (i.e. $L_c << L_{RB}$).

2. The average Rayleigh scattering section is much smaller than the coherence length (i.e. $L_{RSS} << L_C$).
3. The bit duration is much smaller than the roundtrip delay from an individual scattering center (i.e. $T_s << T_{RT}$).
4. The intensity variations due to fiber attenuation, α, are negligible over the bit length (i.e. $T_s << 1/(\alpha v)$) where v is the speed of light in the fiber.
5. The low pass filtering response of the fiber is much lower than the data rate (i.e. $BW_{LP} << R$).

For common telecommunication applications, the above assumptions hold nicely. For example, typical DFB lasers have linewidths around 1 to 10 MHz, installed fiber has sufficiently low loss resulting in effective RB lengths of 20-30 km and data rates commonly exceed 10 Gb/s which many orders of magnitude larger than the kHz low pass filter response of the fiber.

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